

# The High School Mathematics Solution

by Craig Hane, Ph.D. aka Dr. Del

## Dr. Del's Teacher's Guide



# "How to Deliver a Great High School Mathematics Education to Your Students"

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## 1. What's in it for me? (you? your student?)

Welcome.

I assume you are a homeschool high school math teacher, or aspire to be one.

Or, maybe not.

Maybe you just need someone to teach math to your student.

If so, You have come to the right place.

I'm confident I can help You in either case.

Here's what's in if for You.

1..... I will share with you some things I have learned over the years that I believe are crucial in being a good high school math teacher.

2..... You will learn about the Triad of Effective Math Education which is the foundation of all you should do as a math teacher. You will also learn about SPIKE, the Five Ingredients of great pedagogy.

3..... Equally important, I will share with you the content that will be appropriate for your students to lay a foundation for them and . . .

4..... More importantly, get their psychology right to launch them into their further math studies.

5..... As you will learn, you actually have many advantages over a high school classroom math teacher.

However, first a caveat.

In order teach math you must know math.

6..... It probably will not be sufficient to just choose some math textbook and try to guide your student through it.

7..... As you may have already learned many math textbooks are not ideal for teaching math for a variety of reasons. However, I will not critique them here. (See my book, *Teaching Math*)

8..... Rather, I will tell you about the very best high school math book that exists today for Algebra, Geometry, and Trigonometry.

9..... IF you do not really know high school math very well, I will give you a recommendation that just might prove to be your "life saver".

10..... There are several videos which compliment the contents of this Guide. I would encourage you to watch them when reading the Guide.

**11..... The "BIGGY"      If you learn and master the contents of this Guide you will be able to provide your students an excellent high school math education.**

**Let me assure you that thanks to modern technology you will be able to provide your students an excellent high school math education at a reasonable cost regardless of your background or skills.**

What, if any, resources you will need or want to purchase will depend greatly on your own knowledge and skills and ambitions.

However, this Guide should help you with these decisions.

First, let me introduce myself.

*Or, Skip to page 6, and get on with it.*

## 2. Who is Dr. Del?

Delbert Craig Hane, Ph.D. (1967 Indiana University, Mathematics)

Dr. Del is the name his students call him.

Born in Greencastle, Indiana.

Attended Greencastle High School (GHS - 1956)

Studied at DePauw University while in high school. Tutored his fellow students in math and taught trig identities in the GHS trig class.

B.A. Oberlin College, (1960, Math and English). Taught students math as a problem session instructor and tutor.

Taught high school math at Western Reserve high school in Ohio.

Taught math at DePauw while math dept. chairman, Dr. Clint Gass was on sabbatical.

Ph.D. Indiana University, 1967, Algebraic Number Theory.

Taught calculus and logic as a teaching associate and tutored.

Taught three years at Indiana State University (a teacher's college). Taught all of their advanced theory math. Became very disappointed in the state of "math education" for high school teachers. It's only gotten worse since then.

Taught four years at Rose Hulman Institute of Technology, (an engineering school) Math Theory plus calculus and Diff. Eq.

Founded Hane Training and taught technical subjects to adults in many large companies such as Ford, Caterpillar, G.M., State Farm, Navy etc.

Delivered millions of dollars of on-site workshops, and trained many other instructors. *Without exception all of them had to learn more practical math than they already knew.*

**Discovered that the key to success for most of these technical jobs was a proper grounding in practical math.**

This is where Dr. Del really learned how poorly prepared many of our adult workers are in math. He also learned that *almost all of these people are capable of learning practical math when it is presented appropriately.*

A common comment was: *"Why didn't someone teach me this twenty years ago. I had no idea math was so easy."*

You may visit [www.hanetraining.com](http://www.hanetraining.com) to see some of these workshops. They are mostly technical subjects used by maintenance people in a wide variety of jobs such as electronics, electrical power, hydraulics, mechanical power, rigging, PLCs, CNCs, etc.

***The common denominator of all of these subjects is the Practical Math Dr. Del teaches in his Foundation. He says:***

"I love to teach math, both theoretical and practical. However, for most people the best way to begin is with practical math. Theory should only come later, if ever, for most people.

I finally developed a practical math workshop which has been very successful for the students. And, this has spawned an online program that has been very effective with some home school students.

Most students can learn all of the Algebra, Geometry, and Trigonometry, plus how to do all their calculations with a Scientific Calculator (I use the TI 30XA) to solve almost any practical math problem they will ever encounter in any job in about 30 to 50 hours of their time.

Short interactive video sessions with lots of focused problems. Much feedback. Lots of practice and fun. It's truly amazing. You almost have to witness it to believe it. But, it's true. It's like watching a butterfly emerge.

Students go from fear and loathing of math, to enjoying it and practicing it with self confidence. I have seen this in students from ten years of age to sixty years of age. Math is easy and fun IF you teach the right topics with the right methods."

### 3. Triad of Effective Math Education

One of the things I learned over the years is that there are three essential ingredients in a successful effective math education program.

#### I. Psychology. (Job One)

The students' of course. They must lose fear and gain self confidence. Believe it or not, anyone who understands basic arithmetic and can play video games or card games can learn practical math.

However there is a caveat. Until you establish the proper psychology all else will fail. This was always my biggest challenge with adults. Fortunately, it can be corrected in just a few short sessions.

#### II. Pedagogy. (The Process)

It is critical that math be taught properly for these people. A gifted student can learn from any teacher who knows math, or from a good book.

But, most people must have a teacher who meets them where they are and presents the material in a manner they can absorb and understand. Two major challenges for any teacher.

Lectures don't cut it. They can't. There must be interactivity and feedback and self pacing. These three things, particularly self-pacing, are very difficult to achieve in a classroom with a large group of students.

Mistakes should be celebrated as evidence of effort and activity.

Always praise. Of course, explain their mistakes constructively.

Acknowledge all of their successes, even the small ones. Nothing succeeds like success, and breeds confidence, and enjoyment.

**In 2012, I developed a more in depth explanation of how to achieve good pedagogy, which I call **SPIKE – Five Essential Ingredients to a Successful Math Education.****

You may learn all about it at: <http://craighane.com/spike/>

### III. Proper Content. (The Substance)

#### *Content is really critical.*

This is where some math courses really fall down. Some textbooks make the situation even worse.

To learn practical math, which includes appropriate topics from algebra, geometry and trigonometry, plus how to use a scientific calculator for all the calculations, you must select your content carefully.

Proper content selection will result in a much more efficient and effective learning experience, and success.

Many standard math courses include much content that actually detracts from this.

For example, unnecessary theory should be deferred. It is unconscionable how many beginning math students are turned off by their math studies due to theoretical materials being forced on them prematurely. Rigorous proofs must wait for adequate maturity.

Topic selection is also quite important. One should not present advanced topics prematurely either. This again leads to discouragement. It amazes me how often this happens too.

You may learn more about the Triad including a video at:

<http://www.triadmathinc.com/triad/>

Just remember:

The student's psychology is determined and affected by both:  
How you teach and What you teach.

You must present a topic to a student only if he or she is adequately prepared for it.

The best foundation for your student is the content that is presented in the Dr. Del Practical Math Foundation. You will be given all of this information below.

#### **4. Attributes of a good high school Math teacher.**

This is where the rubber meets the road.

There is no "one size fits all".

My two best teachers, without whom I would not have succeeded in math, were an "old maid" high school math teacher and a very reserved college math professor. I thank God for Miss Madonna O'Hair and Dr. Clint Gass. They "saved" me.

I disliked math until I met Miss O'Hair at GHS in the tenth grade Geometry class. I had hated algebra my freshman year. Then, Dr. Gass, at DePauw, propelled me into a career in math with a first course in College Algebra when I was a senior at GHS.

I subsequently survived many weak teachers. All, it takes is a good foundation and confidence in one's own abilities. Without those two teachers there would be no Dr. Del, or his Practical Math Foundation.

Now then, there are several attributes which I believe are necessary to be a successful math teacher.

Listen to and be empathetic to your student.

Your student's psychology is the very first thing you must understand and be aware of. The best way to determine this is to ask questions and listen to the answers. Clearly, you must ask the right questions, and this is somewhat of an "art".

But, generally, I just ask the student things about math that is prerequisite to what I am going to teach them. IF they don't know these prerequisites, then I back up and cover those topics before proceeding. NEVER skip this step.

Encourage the student. Praise all effort, even when it is not producing desired results. Celebrate mistakes as evidence of effort and activity.

Of course, you want to constructively help the student discover his or her mistake and help them to correct it.

I treat math like a sport or a game like chess or cards. You must “do it”. You must practice. You will make a lot of mistakes. And, that is good. That is how you learn and develop your skills. In a sense, a math teacher should be like a coach.

The learning of math should be “self paced” and “interactive”.

Each student learns at his or her own pace. This is natural and fine.

Each student needs to be “interactive” with the instructor.

Students need repetition of certain things. Some more than others.

The best way to achieve this is with a very patient tutor. This is sometimes impossible to obtain and/or afford.

*Lectures just don't cut it for most students. That is why most so many times classroom teaching falls short.*

It is better to pretend you are learning the topic for the first time and ask a series of questions that lead you and the student through a sort of discovery process.

It is OK to also go down a wrong path sometimes and then realize you are at a dead end. That is how all research mathematicians operate.

It is OK, No, more than OK, to make mistakes. That is how real math is done. Like any meaningful sport or game or job.

And, it should be enjoyable, mistakes and all. I like to chuckle a lot at my mistakes. So, should your student. Have fun.

You will be most effective for the student when it seems to the student you are learning along with him or her. Indeed, in a sense you should be. I understand a math topic better each time I teach it.

That is how I got started as a math teacher. I was in high school and teaching my peers. So, I really was learning along with them.

Being a good math teacher is very similar, and just as difficult or easy, as being a good coach.

Do not think that you can just buy some textbook and make assignments and grade problems. That will not suffice for most students. It's a recipe for failure.

When the student gets frustrated or discouraged try to use humor. And, remind her of how well she did at some point in the past.

Math should be fun. Just like a challenging game.

**If all of this seems impossible or too much trouble for you then I would recommend you get someone else to teach your student math.**

Of course, you can hire a tutor or go to a special school like Sylvan. But, that may not be feasible or affordable for you.

*Fortunately, there are some resources available today that weren't available even a few years ago.*

*See our Resources section.*

**In the next section I will discuss one such resource, the Practical Mathematics Foundation, which you may use as a benchmark to evaluate whatever you choose to do.**

You may also use its Syllabus as a Lesson Plan if you are going to be the math teacher. You will be given this at our website

<http://www.triadmthinc.com/the-foundation-course/>

You might even want to take the Foundation yourself to become a better math teacher.

**Anyone who completes the Foundation can probably do a pretty good job helping another student learn Practical Math.**

If you have a series of students to teach, you might want to learn it the first time along with your first student, and then the two of you teach the others later. It will be a wonderful experience for both you and your oldest student. It's how I learned math. Teaching it to others. Remember a new topic is like a discovery to a student.

## 5. The Dr. Del Practical Math Foundation

You may visit <http://craighane.com/pmf/> for a full explanation of this.

But, here is a copy of an article on it which I will discuss for you.

My comments will be given in [ *Comments in Italics* ]'s.

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### **Dr. Del's Practical Math Foundation**

No matter what your past experiences with math have been,

NOW YOU CAN . . .

**"Learn to Solve any practical Math Problem**

**You are likely to Encounter — Quickly and Easily"**

Beginner? – Experienced? – Skilled Tradesman?

Worker? – Student? – Teacher? – Parent?

Plumber? – Laborer? – Carpenter? – Mechanic?

Electrician? – Landscaper? – Chef? – Scoutmaster?

Math phobia? – Hate math? – Like math?

Learn what you need to know about Algebra, Geometry, and Trigonometry, plus how to use a scientific calculator for all your calculations. NO unnecessary theory or topics.

Fun . . . Easy . . . Fast

*[I have helped people in all types of jobs and projects solve their math problems for decades. And, it would be a very rare everyday practical problem that I would have needed something not included in the Practical Math Foundation.*

*However, a very important **unstated goal** of the Foundation is to "recalibrate" the student's psychology. Lose all fear and dislike of math and learn to enjoy math.*

*The Foundation will teach all the practical math needed in ordinary real world situations. But, it also will get the student ready for further math studies should they want to go on in science or engineering or some other technical subject. See the Ten Tiers below.*

*The Foundation provides a type of foundation for both jobs and further study. I would put ANY student through it before any other math studies. You might want to study the Ten Tiers chapter.]*

Online self-paced video training with problems takes most people about forty to fifty hours of time to complete. Plus, you may participate in a Forum and ask any questions. Dr. Del monitors the Forum and answers any questions your fellow students can not answer for you.

The best way to learn is to work problems and help others too.

*[This is really important. It's like a sport. We learn by helping and encouraging each other. The best way to learn a math topic is to teach it to someone else. After, you have mastered it yourself, of course.]*

Your time is money.

Learn Practical Math very quickly with a minimum of your time invested.

*[This is really important. The Foundation should take your student about 40- 50 hours spread over two or three months. There is no good reason to stretch out any longer the first pass through these three subjects.*

*These three subjects need to be integrated. Rarely do I solve a real world practical math problem that does not involve at least two of the three subjects. Students need to realize this early in their studies of math. See example below on page 15.*

*This theme then needs to be continued for those students who choose to go on beyond the Practical Math Foundation. See the Ten Tier Chapter for a discussion of this.]*

Most people complete the Foundation and master practical math in about thirty to fifty hours of their time spread over 1 -3 months.

However, there is no hurry or time limit. Your study is self paced. Study at your leisure. Take as long as you need. This training is interactive with feedback.

FUN. Practical Math will be fun for you. It is as easy as learning a new game or sport. And, . . .

*[This is very important. Each of us learns at our own pace. This is necessary for us to gain the joy we should from doing math. And, having FUN is just as important in math as any other human activity such as drama, music, writing, or sports.]*

Just Imagine the Value of being a “math expert” in your job or hobby.

Also, Imagine the effect learning practical math will have on your test scores for most standardized tests. I can assure you it will be a dramatic improvement.

*[This is very important. Just imagine being in the upper 5% of your fellow citizens in math knowledge and ability. Think how much this will help you in many jobs and in your self confidence in your abilities.*

*Math is a great adjunct to almost any quantitative or technical job or subject. Math competency can be just as valuable as language competency in many situations.]*

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I discuss all of these issues at greater length in my book

***21<sup>st</sup> Century Mathematics***

## ***Teaching Math***

**“A High School Math Curriculum That Works for All Students”**

Available at Triad Math, Inc. [www.TriadMathInc.com](http://www.TriadMathInc.com)

Get a FREE PDF version of ***Teaching Math*** at:

[www.triadmathinc.com/tg/](http://www.triadmathinc.com/tg/)

## 6. Syllabus or Lesson Plan

If you are going to teach math to your high school student, you should start with a Foundation type of program.

The Practical Math Foundation includes, How to use a scientific calculator and Pre-Algebra (Tier 1), and Algebra, Geometry, and Trigonometry (Tier 2)

If you know math, then you should be able to teach these topics to your student very easily.

Here is a Syllabus of Dr. Del's Foundation that can serve as a lesson plan for you. <http://www.triadmthinc.com/foundation-syllabus/>

You should be able to teach each of these subjects to your student at your kitchen table too, like Dr. Del does in the videos.

Note that learning math is kind of like climbing a ladder. You need to do it in the proper sequence one rung at a time. This Syllabus/Lesson Plan should help you.

Dr. Del does not use a textbook for this Foundation. He does use a set of notes he has prepared for the student which go with each lesson or topic.

You may find the syllabus or list of topics at:

<http://www.triadmthinc.com/foundation-syllabus/>

Keep in mind that the student is expected to pause the videos and repeat when needed. So, they are to be self paced.

Thus, it will take you longer for each topic, and you need to be patient, very patient, with your student. A five minute video might take twenty minutes or longer in a live tutorial setting. Go at your student's pace, NOT yours. Do not rush the student.

**If you feel you are not ready to teach all of these subjects, then you might want to take the Foundation yourself and then present the material.**

**That is, you could make yourself a better math teacher by learning from Dr. Del. That would delight him.**

**Here's An Example to demonstrate the interconnectedness of all three subjects, Algebra, Geometry, and Trigonometry.**

As mentioned above, it usually takes a combination of two or three of these three subjects to solve a math problem. Here is an example.

Find the area to within .01 sq. in. of a triangle whose three sides measure: 3.00 in, 4.00 in, and 6.00 in. The answer is 5.33 sq. in.

After completing the Foundation a student should be able to solve this problem in a minute or so.

**So, think about it and see how you would solve it. You will appreciate this example much more if you do.**

Most people remember the formula for the area of a triangle as:

$(1/2) \times \text{Base} \times \text{Height}$ , which is true, of course.

But, it doesn't help much here since I didn't tell you what the height was in any direction. So this formula can't be applied directly.

Of course, if this was a right triangle it would work, but it isn't.

That's the first thing one should check with the Pythagorean Theorem.

But,  $3^2 + 4^2 = 25$  and  $6^2 = 36$ . So it's not a right triangle.

What is required for this problem is a combination of algebra, geometry, and trigonometry. And, it is a real shame for a student to have to study three years of math to learn how to solve it.

Similar situations arise with many real world problems. So here is how a Foundation student learns to solve it.

One solution we teach in the Foundation Course:

Since the triangle is not a right triangle we use the more general formula we learned in geometry (Triangle video) for the area when we know the measure of two sides  $a$ , and  $b$ , and the included angle  $\angle C$ .

Area =  $.5 \times a \times b \times \text{SIN}(\angle C)$  [ $b \times \text{SIN}(\angle C)$  is Height to Base  $a$ ]

But, what is  $\angle C = ?$

Now, we need to use what I call the Generalized Pythagorean Theorem commonly called the Law of Cosines.

Note:  $c$  is the length of the side opposite  $\angle C$ .

$$c^2 = a^2 + b^2 - 2ab\cos(\angle C)$$

$$c \times c = a \times a + b \times b - 2 \times a \times b \times \cos(\angle C)$$

$$\text{So in our problem } 6^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \times \cos(\angle C)$$

A little algebra yields:

1.  $\cos(\angle C) = (9 + 16 - 36)/24 = -.4583333333$
2. Thus,  $\angle C = \text{INV COS} (-.45833333) = 117.28$  degrees
3.  $\text{SIN}(117.28 \text{ degrees}) = .88878$
4. Thus,  $\text{Area} = .5 \times .88878 \times 3 \times 4 = 5.33$  sq in.

This is much easier to explain with a Sharpie and TI 30XA at the "Kitchen Table" and is a problem that a student can do with a little practice in a couple of minutes.

And, calculating the Area of a triangle with sides 7.4, 8.5, and 12.8 is just as easy since we are using a calculator.

ANS: 30.07 in less than one minute. Checked answer with two different sides in less than one minute also. In the Foundation we learn to Always check our answers.

In fact, I (you) can teach a young person to do this as a four step process in several minutes with them practicing several examples. In fact, I find it takes a typical student about ten examples to really master it, the "learning curve". Then, they can challenge others with a similar problem and take pride in their new found skill. A real confidence builder.

Hey! Learn it yourself, and then teach a student!

## 7. Importance of the Scientific Calculator

Prior to 1972 in order to solve practical math problems a person had to learn a wide variety of algorithms, i.e. techniques for calculations.

Some of these were pretty easy thanks to the marvelous invention of the positional Base 10 decimal number system. Everyone learned algorithms for such things as addition, multiplication, and division.

After several years of practice most people were reasonably proficient.

Nevertheless, it was time consuming, boring, and mistake prone.

Checking one's work took as long as the original calculations. Ugh.

Just imagine how long the problems above for the area of a triangle would take to solve manually.

There were tools to help like something called a "slide rule". This was based on logarithms which were another indispensable calculation tool. Of course, then one needed look up log tables.

Then there were trig tables. Things like square root extraction were more difficult. Higher roots required logarithms.

In fact, in order to learn practical mathematics one had to learn to do the various complicated and tedious calculations which were laborious and took a long time to learn.

I would estimate that one easily spent 80% or more of one's time learning and carrying out calculations.

*Then a "magical" thing happened.*

The scientific calculator was invented.

The HP 35 was the world's first electronic scientific calculator. It cost about \$2,243 in 2014 dollars. That was 1972. It had 35 Keys.

Guess what? That revolutionized the way calculations were done.

Math problems that would take one hour to solve now could be done in about ten minutes or less. Try the above problem manually.

Slide rules became obsolete overnight.

The material that you will learn in the Practical Math Foundation would have required two or three years to learn and involved many difficult algorithms and look up tables and logarithms and so on.

Today the TI 30XA is a much more powerful calculator than the HP 35. And, it is much easier to use. And, it costs about \$10 in 2014.

Like any powerful tool, it takes some time to learn to use it. But, as you will learn in the videos on the calculator in the Foundation it can be mastered for Practical Math in a few hours.

Then it will save you countless hours of your precious time.

You will not have to master complex algorithms. You will not need books of look up tables. You will not have to learn to use a slide rule.

You will be able to do complex calculations in a few minutes and check your work just as fast.

It will make the learning of Algebra, Geometry, and Trigonometry as they apply to practical math problems much much easier.

It is the reason the Foundation will require only 30 to 60 hours of your time depending on your aptitude and efforts.

To learn this much math without using the calculator would probably take you at least TEN times longer. And, it wouldn't be very much fun grinding through the calculations.

Anyone who refuses to use a scientific calculator in the teaching and application of practical math is simply ignorant or stubborn. It would be like teaching or doing carpentry with only hand tools. Very foolish and wasteful of time.

No modern employer would pay any employee to perform old fashioned manual calculations.

Any teacher who fails to utilize a calculator in the teaching and performance of math will do a great injustice to their student.

## **8. Beyond the Foundation -- Ten Tiers**

So, your student has successfully completed the Practical Math Foundation, Algebra, Geometry, and Trigonometry – First Pass.

What next?

For some students, there is no next. They are ready for most real world jobs. They may choose not to go on to further math studies.

However, for those students who might want to pursue a STEM career or educational path, there is much more to come.

We are creating a comprehensive Ten Tier program.

The first Six Tiers are all any student would need to matriculate into the best colleges and universities in the country to pursue a STEM career.

The last Four Tiers are for those rare students who are both mathematically gifted and who might contemplate a career in mathematics.

Here are descriptions of the Ten Tiers. The first Six Tiers should take most students about three years. There will be one wonderful \$20 book required. See below (Simmons).

It will be used in Tiers 3 and 4. This is the closest we get to a conventional high school curriculum.

Still it is quite different, and much superior.

In the upper Tiers we use special notes and videos exclusively just like we did in the Foundation.

We were just very fortunate to have the wonderful book from Dr. Simmons as a resource for two of the Tiers, actually about one and a third.

The first Six Tiers are a very good model for you if you choose to do your own teaching.

## **“A Comprehensive Ten Tier Mathematics Program for High School Age Students”**

**Prerequisites:** A basic knowledge of the decimal number system and a familiarity with basic geometric figures such as triangles, rectangles, and squares. The kind of math any student should have by the end of the seventh or eighth grade. Some students are ready by seventh or eighth grade.

Note: It is not necessary for the student to have mastered any particular algorithms. We will use the calculator extensively for important pedagogical reasons discussed above.

The Ten Tiers will then be as follows:

### **Tier 1. Learning to use the Scientific Calculator, the TI 30XA.**

This series of video lessons has the objectives of getting the student familiar with and comfortable with using the scientific calculator.

It is not intended to teach very much math at this point in time. We will explain the basic arithmetic functions and review number concepts, but not explain any of the more advanced functions such as the trig functions.

Yet, we will have the student become familiar with most of the keys. They will learn to fix the decimal to so many places, to use the memory and parentheses, fractions, and some of the special function keys, etc.

We will not discuss keys not used in the Foundation such as log, or hyp trig, or statistics keys.

The goal is to get the student started learning math and comfortable with the calculator. And, perhaps to have a little fun.

Then we have lessons on Pre-Algebra to review the rules of arithmetic and get the student even more “oiled up” to do the arithmetic.

Homework Exercises and online Quizzes are use to help both the student and mentor assess progress and recognize achievement.

## **Tier 2. The Dr. Del Practical Math Foundation**

The student will be presented what we call Practical Math, which includes the topics from Algebra, Geometry, and Trigonometry that are needed for ordinary practical everyday math problems.

No theory to speak of, few proofs, not too many definitions. Just the basic math useful for everyday life in almost any activity.

The student will be taught through a series of videos and problems sets. Virtually all students, regardless of aptitude and ability, should be able to successfully complete this course. The average student should take 30 to 60 hours spread over one to three months to complete this course.

Some students may choose to stop here depending on their educational and career aspirations. At this point they will know more math than about 95% of the U.S. population. And, be well prepared for tech school or many real world jobs.

## **Tier 3. Geometry I, Algebra I, and Trigonometry I ala Simmons**

For students who wish to excel on the mathematics part of the SAT or ACT tests, but who may not pursue a STEM career. Tier 3 is a more conventional high school curriculum.

Algebra, Geometry, and Trigonometry will be presented in a series of videos on each topic with exercises from the book, and online quizzes to reinforce the student's progress. There will also be some topics with special notes.

Dr. George Simmons book, *Precalculus Mathematics in a Nutshell* will be the text for these three courses. It is a fabulous book. \$20 or so.

Dr. Del will be the teacher presenting these materials.

**After Tier 3, the student will be prepared for doing well on the SAT and ACT, but will need more math for calculus as taught at any first rate college or university.**

**Thus, there are more topics covered in Tier 4, which would yield a truly first rate precalculus preparation.**

#### **Tier 4. Geometry II, Algebra II, and Trigonometry II plus Complex Numbers and other topics.**

We leave about 60% of the more difficult material in Simmon's book until Tier 4. Math needs to be taught in a Tiered way to let the student develop his or her "Mathematical Maturity".

This will be math presented to the student who wants a deeper understanding. It will include topics from so-called Finite Mathematics such as Probability and Statistics, Logic and Set Theory, Functions and Relations, and a few other topics..

More theorems will be proven.

The incredible tool, Wolfram-Alpha, will be introduced and utilized. This 21<sup>st</sup> Century tool literally revolutionizes the learning and practice of many math topics starting with Algebra. It totally revolutionizes the practice and learning of calculus and linear algebra.

***Complex numbers will be explained and their relationships to many other mathematics subjects discussed. This is quite important and unusual at this stage of math education, but very essential for a future STEM career.***

***This really is a BIG DEAL. You probably need to be a mathematician, or engineer or scientist, to appreciate this.***

*Now the student is really ready for calculus. Not only in terms of prerequisite material, but also in their math maturity. Tiers 1 – 3 should take most students about one year, and then Tier 4 another year.*

#### **Tier 5. Calculus**

Calculus will be explained conceptually, and the great software tool Mathematica introduced. Mathematica is almost a "miracle" tool. It does for Calculus and many other advanced math subjects what the scientific calculator does for ordinary elementary calculations.

This will not be “your father’s” course in calculus. Much Better.

It will be great preparation for a university course in calculus, and also good preparation for science, engineering, or technology. Probably all that will be needed for engineering.

All too often the important concepts of calculus are obscured by the myriad details covered in a typical course. Much of the traditional materials taught in a typical calculus course are mitigated by tools such as Mathematica.

### **Tier 6. Differential Equations**

Differential Equations will be introduced, again with Mathematica as the power tool. This will be a conceptual treatment that will prepare the student for a more conventional university course.

**Tiers 5 and 6 together will take about one year for most students.**

**All but the most precocious students will stop here, and probably matriculate on to university.**

**They will be better prepared than all but a very few freshman.**

However, for the students with a high aptitude for math and a strong desire to learn more before going on there are four more Tiers.

### **Tier 7. Basic math theory and concepts**

Advanced theoretical math will be discussed starting with a discussion of the various number systems, the axiomatic approach, math models, equivalence relations, indirect proofs, Axiom of Choice/Zorn’s Lemma, Mathematical Induction, Fractals and non-integral dimensions, etc.

Many topics will be discussed in preparation for the last three Tiers. These Tiers normally wouldn’t appear until late in college math.

### **Tier 8. Math’s basic structures and techniques**

Some of the most important basic theoretical mathematical structures and techniques will be introduced.

Building math structures from existing math structures using such techniques as cross products and then quotient structures.

For example, building the real numbers from the rational numbers, and then the hyperreals from the reals using ultrafilters.

Quotient structures are often a major conceptual hurdle aspiring mathematicians must cross. Once this is mastered the student will be ready for more advanced theoretical math courses at university.

### **Tier 9. Modern theoretical math structures.**

More advanced abstract math structures will be discussed.

Many of the math structures used by mathematicians will be introduced. Topological Spaces, Groups, Rings, Fields, Hilbert Spaces, Measure Theory, Linear Algebra, Modern Analysis will all be discussed from an overview conceptual point of view.

Chaos, Non-linear system, Catastrophe Theory will be discussed.

The purpose of Tier 9 is to give the student an overview understanding of modern math to help determine what directions he or she might wish to pursue.

And, to open their mind to the possibilities. Most people are totally unaware of what modern math consists of.

### **Tier 10. Infinity, Godel, Spectral Logic, Model Theory, etc.**

This could come before or in place of Tier 9.

Concepts of Infinity and Godel's Theorem and other ideas will be discussed.

Model theory and the fascinating story of infinitesimals will be told.

How math models apply to science. Some brief discussions of some math models such as those used in Quantum Theory and Relativity Theory.

Spectral (or Fuzzy) Logic will be discussed.

**By the end of Tier 10, a student should have a very good idea about what mathematics is all about and how much more they might wish to learn.**

**Most students are lucky to get this much insight into modern math by the end of a math B.A. degree.**

**However, precocious students with a high interest level in math should be given this by the end of their high school studies.**

**At least, that is my opinion. And, I know it can be done. The brightest students in the best schools may get this now, but it is quite rare.**

**Tiers 7 – 10 will be enough to challenge the brightest students for as long as they have until they go off to university. These students may complete Tiers 1 thru 6 by the end of their sophomore year.**

**That obviously gives them a couple of years to study until they finish high school. They may very well need to not matriculate to university early due to social maturity or other subjects.**

**However, they will be able to pursue their math studies very productively for these two or three years.**

**I have seen precocious students in math go off to university too early with unpleasant results.**

**IF you have a child or student who is precocious in math and really likes math, then this will be invaluable to him or her.**

**BTW, I have found that girls are just as good in math as boys. In fact, my daughter has a Ph.D. in computer science and is a renowned professor.**

## **9. Resources for You recommended by Dr. Del.**

### **I. PreCalculus Mathematics in a Nutshell (\$20 or so)**

**by Dr. George Simmons**

**Geometry, Algebra, and Trigonometry all included**

**This is an outstanding book for any high school math teacher.**

**It is only 120 pages. It includes virtually all topics in these three subjects plus examples and problems and supplemental materials.**

**It is condensed with no fluff, yet very complete. Simply fabulous. You must get it.**

**Proofs are elegant and complete.**

**I use it in Tiers 3 and 4. We pass through it twice. This helps the student gain mathematical maturity. This is hard to define, but any good math teacher knows what it is. Math professors talk about it all the time.**

### **II. The TI 30XA Scientific Calculator (\$10 or so.)**

**Another fabulous tool. A marvel of modern technology.**

### **III. The Kahn Academy <http://www.khanacademy.org/>**

**A terrific resource of short videos on a very wide variety of subjects including many topics from math.**

**Excellent for teachers or students who know what they want to learn about next. Bill Gates favorite resource we are told.**

**This is the first place I go if I want some specific information on any topic Sal Kahn has created a video on.**

**IV. Wikipedia [www.wikipedia.org](http://www.wikipedia.org)**

**Enough said. You can get definitions and information on just about anything. This material is written usually.**

**Again, you have to know what you are looking for.**

**But a fabulous resource if you know what you are looking for.**

**V. Dr. Del's Practical Math Foundation**

**<http://TriadMathInc.com/>**

**This is not a reference source like III and IV above.**

**It is much more of a comprehensive integrated program.**

**It is a program where the student is led through a specially selected series of topics with appropriate problem sets, and quizzes, and a forum for participation with other students and Dr. Del on occasion.**

**You don't need to know what you are looking for as in a reference source. Dr. Del makes that decision for you. As he should.**

**Most homeschool teachers don't know enough math to know what is appropriate and are at the mercy of whatever program they choose to use. So they better choose a program that is good for their students.**

**This is an excellent resource for any home school teacher who is not a math expert. There are two choices then.**

**The teacher can learn math along with their student and thus become a much more effective teacher. OR**

**This is not necessary. This is a complete resource for the student where the teacher can be more of a mentor and**

**cheerleader. You can let Dr. Del be your student's math teacher.**

**This Foundation should take only a few months to complete.  
VI. Dr. Del's Ten Tier System, Tiers 3 thru 10.**

**This is not open to the public yet. It is under development and being used by a few "early adopters". Tier 3 has just recently been released. Visit: [www.CraigHane.com](http://www.CraigHane.com) for details.**

**This is open only to students who complete the Foundation.**

**It will essentially be a membership site. The prices are yet to be determined. But, Dr. Del is committed to having no worthy student denied this because of lack of funds. So scholarships will be available.**

**This will be the most outstanding high school math education available anywhere in the world today.**

**But, first things first.**

**All students must go through the Foundation before they will be accepted into the higher Tiers.**

**That is for the student's benefit.**

**And, everything Dr. Del does is for the student's benefit. Not anyone else's. Well, of course, the parent benefits too.**

**But in the long run, the student is really Dr. Del's only "customer".**

**He really doesn't care what anyone else thinks one way or the other as long as the student benefits.**

**As it should be.**

**Do you agree?**

## **FINAL ADVICE**

**Parent or student.**

**Try the Practical Math Foundation.**

**Evaluate it for yourself.**

**If it works for you, then use it and benefit from it.**

**Either as a teacher or as a student.**

**Of course, there is a 100% Money Back Guarantee if for any reason you try it and don't like it.**

**So, when you and/or your student is ready just go to the Foundation button at [www.TriadMathInc.com](http://www.TriadMathInc.com) and Order the Course.**